

Quantum Nucleation in a Ferromagnetic Film Placed in a Magnetic Field at an Arbitrary Angle

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Abstract

We study the quantum nucleation in a thin ferromagnetic film placed in a magnetic field at an arbitrary angle. The dependence of the quantum nucleation and the temperature of the crossover from thermal to quantum regime on the direction and the strength of the applied field are presented. It is found that the maximal value of the rate and that of the crossover temperature are obtained at a some angle with the magnetic field, not in the direction of the applied field opposite to the initial easy axis.

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Consider a ferromagnetic film of thickness z_t and its plane is perpendicular to the easy axis determined by the magnetocrystalline anisotropy energy depending on the crystal symmetry. Since the magnetic anisotropy energy has a time reversal symmetry in the absence of an external magnetic field, \mathbf{M} has at least two easy directions. By applying a magnetic field \mathbf{H} in a direction between perpendicular and opposite to the initial easy axis along which the system is magnetized, the symmetry of two equivalent orientations of \mathbf{M} is broken. According to the classical theory of nucleation, [1] the switching of the magnetization occurs for a magnetic field greater than the critical field H_c at which the spins of a saturated ferromagnet cease to be aligned. Besides the magnetic anisotropy energy E_a , the exchange energy E_{ex} and the magnetic dipole-dipole interaction E_{dd} determine the switching process of the system. In this paper we simplify the system with small E_{dd} compared with E_a and E_{ex} . [2,3] At finite temperature, even for the field less than H_c , there is a nucleation due to thermal fluctuation whose rate is proportional to $\exp(-U/k_B T)$ where U is an energy barrier related to the magnetic anisotropy energy, the external magnetic field and the exchange energy. At a temperature low enough to neglect the thermal activation process, the magnetic nucleation which is independent of temperature, may occur due to quantum tunneling. For the dynamical situation, it is important to include the effect of the environment on the quantum tunneling rate caused by phonons, [4] nuclear spins, [5] and Stoner excitations and eddy currents in metallic magnets. [6] However, many studies have shown that while these coupling might be crucial in macroscopic quantum coherence, they are not strong enough to make the quantum tunneling unobservable.

The question of the quantum nucleation were studied by Privorotskii [7] who suggested the WKB exponent based on the dimensional analysis. Later, Chudnovsky and Gunther [2] proposed the quantum nucleation in a ferromagnetic thin film at zero temperature by using the instanton technique. In their work it was considered the system with the external magnetic field applied in the opposite direction to the initial easy axis, and presented the rate of quantum nucleation which allows experimental observation with a sufficient probability. In this paper we shall extend their considerations to the system with the magnetic field applied

in a range of angles $\pi/2 < \theta_H \leq \pi$ where θ_H is the angle between the initial easy axis (z -axis) and the external magnetic field, which, in its method and spirit, is the generalization of Ref. [8] to problems with a non-uniform rotation of \mathbf{M} .

Using the parameter defined to be $\epsilon = 1 - H/H_c$, at zero temperature the nucleation exponent $B_Q \propto \epsilon^{3/4}$ in a wide range of angles, not $\epsilon^{1/2}$ which was obtained by Chudnovsky and Gunther for $\theta_H = \pi$. [2] The crossover temperature T_c for quantum nucleation to thermal activation has a weak dependence on ϵ , $T_c \propto \epsilon^{1/4}$ in a wide range of angle [8], compared with $\epsilon^{1/2}$ at $\theta_H = \pi$. [2] Also, it is found that the direction of \mathbf{H} which favors the quantum nucleation lies in the range of angle $\pi/2 < \theta_H < \pi$, not π . In this paper we shall calculate instantons in the range of angles $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$ at $T = 0$ based on the imaginary time path integral method and their contribution to the quantum nucleation exponent B_Q , and discuss the θ_H dependence of B_Q and the crossover temperature T_c from quantum nucleation to thermally assisted process.

Our considerations begin by writing the transition amplitude in the imaginary time $\tau = it$ from $-\tau_I/2$ to $\tau_I/2$ based on the spin coherent state path integral formalism

$$\langle \hat{z} | \exp(-\tau_I H/\hbar) | \hat{z} \rangle = \int D[\mathbf{M}(\mathbf{r}, \tau)] \exp(-S_E/\hbar), \quad (1)$$

where \hat{z} is taken to be an initial easy direction of the magnetization, and S_E is the Euclidean action which includes the Euclidean Lagrangian density L_E as

$$S_E = \int d\tau \int d^3\mathbf{r} L_E. \quad (2)$$

Since the magnitude of the total magnetization \mathbf{M} is frozen up at sufficiently low temperature, the direction of the total magnetization becomes the only dynamical variable which we are interested in. In this situation the measure of the path integral $D[\mathbf{M}(\mathbf{r}, \tau)]$ in Eq. (1) is replaced by $D\Omega$ as

$$D\Omega = \lim_{\epsilon \rightarrow 0} \prod_{k=1}^N \left(\frac{2J+1}{4\pi} \right) \sin \theta_k d\theta_k d\phi_k, \quad (3)$$

where $\epsilon = \max(\tau_{k+1} - \tau_k)$ and $J = M_0/\hbar\gamma$. Here $\gamma = g\mu_B/\hbar$, with g being the g-factor, μ_B the Bohr magneton, M_0 the magnitude of the magnetization. Using the standard instanton method, the rate of quantum nucleation has the relation

$$\Gamma_Q \propto \exp(-S_E/\hbar), \quad (4)$$

and its Euclidean action for this problem is given by

$$S_E = \int \left\{ i \frac{M_0}{\gamma} [1 - \cos \theta(\mathbf{r}, \tau)] \frac{d\phi(\mathbf{r}, \tau)}{d\tau} + E[\mathbf{M}(\mathbf{r}, \tau)] \right\} d\tau d^3\mathbf{r}. \quad (5)$$

The first term in Eq. (5) is the topological Wess-Zumino term [9], and the second term is the energy density which is composed of the magnetic anisotropy energy, the exchange energy and the energy given by an external magnetic field \mathbf{H} . So, the energy taken into account can be written as

$$E[\mathbf{M}(\mathbf{r}, \tau)] = E_a + E_{ex} - \mathbf{M} \cdot \mathbf{H}. \quad (6)$$

We notice that from $\delta S_E = 0$ the action (5) produces the classical equation of motion [10] whose solution is called instanton, [11] and describes the $1 \oplus 1$ dimensional dynamics in the Hamiltonian formulation, which consists of the canonical coordinates ϕ and $p_\phi = 1 - \cos \theta$. [12,13]

In this paper we study the biaxial symmetry whose magnetic anisotropy energy is given by

$$E_a = K_1(\alpha_1^2 + \alpha_2^2) + K_2\alpha_3^2, \quad (7)$$

where K_1 and K_2 are the parallel and transverse anisotropy constants, and α_1, α_2 , and α_3 the directional cosines. The exchange interaction is expressed as [1]

$$E_{ex} = \frac{1}{2}C[(\nabla\alpha_1)^2 + (\nabla\alpha_2)^2 + (\nabla\alpha_3)^2] \quad (8)$$

where C is an exchange constant. Applying the magnetic field in the xz -plane [8] and expressing α_1, α_2 and α_3 in terms of the fields $\theta(\mathbf{r}, \tau)$ and $\phi(\mathbf{r}, \tau)$ which are spherical coordinates of \mathbf{M} , the total energy (6) is given by

$$E[\theta(\mathbf{r}, \tau), \phi(\mathbf{r}, \tau)] = K_1 \sin^2 \theta + K_2 \sin^2 \phi \sin^2 \theta + \frac{1}{2} C [(\nabla \theta)^2 + (\nabla \phi)^2 \sin^2 \theta] - H_x M_0 \sin \theta \cos \phi - H_z M_0 \cos \theta + E_0, \quad (9)$$

where E_0 is introduced to make $E(\theta, \phi)$ zero at the initial orientation. As will be seen later, the effective mass of the magnetic particle is inversely proportional to a linear combination of K_2 and H_x while there is no exact analog of the kinetic energy in the action (5). For this reason we need either the transverse anisotropy constant K_2 or the magnetic field H_x transverse to the initial easy axis for quantum nucleation. Introducing the dimensionless constants,

$$\bar{K}_2 = K_2/2K_1, \quad \bar{C} = C/4K_1, \quad \bar{H}_x = H_x/H_0, \quad \bar{H}_z = H_z/H_0, \quad (10)$$

where $H_0 = 2K_1/M_0$, we obtain the total energy (9) written as

$$\bar{E}[\theta(\mathbf{r}, \tau), \phi(\mathbf{r}, \tau)] = \frac{1}{2} \sin^2 \theta + \bar{K}_2 \sin^2 \phi \sin^2 \theta + \bar{C} [(\nabla \theta)^2 + (\nabla \phi)^2 \sin^2 \theta] - \bar{H}_x \sin \theta \cos \phi - \bar{H}_z \cos \theta + \bar{E}_0, \quad (11)$$

where $\bar{E}(\theta, \phi) = E(\theta, \phi)/2K_1$. The plane given by $\phi = 0$ is the easy plane, on which $\bar{E}(\theta, \phi)$ is given by

$$\bar{E}(\theta, 0) = \frac{1}{2} \sin^2 \theta - \bar{H} \cos(\theta - \theta_H) + \bar{E}_0, \quad (12)$$

where we used $\bar{H}_x = \bar{H} \sin \theta_H$ and $\bar{H}_z = \bar{H} \cos \theta_H$ since \mathbf{H} lies in xz -plane. Let us define θ_0 to be the angle of metastable state generated by the anisotropy energy and the external magnetic field, and θ_c the angle at which the barrier vanishes by the applied critical magnetic field H_c . Then, θ_0 is determined by $[d\bar{E}(\theta, 0)/d\theta]_{\theta=\theta_0} = 0$, and θ_c and \bar{H}_c by $[d\bar{E}(\theta, 0)/d\theta]_{\theta=\theta_c, \bar{H}=\bar{H}_c} = 0$ and $[d^2\bar{E}(\theta, 0)/d\theta^2]_{\theta=\theta_c, \bar{H}=\bar{H}_c} = 0$. After a little calculations, the dimensionless critical field \bar{H}_c and the critical angle θ_c are expressed as [13,8]

$$\bar{H}_c = (\sin^{2/3} \theta_H + |\cos \theta_H|^{2/3})^{-3/2}, \quad (13)$$

$$\theta_c = \frac{1}{2} \arcsin \left[\frac{2 |\cot \theta_H|^{1/3}}{1 + |\cot \theta_H|^{2/3}} \right]. \quad (14)$$

For, e.g., $\theta_H = 3\pi/4$ and π , we have $\theta_c = \pi/4$ and 0 , respectively. Simple analysis for Eq. (12) shows that, since the height and the width of the barrier are proportional to the power of the parameter ϵ , the value of ϵ should be small in order to have a large tunneling rate. The small value of ϵ can be achieved by tuning H to H_c . From now on we will consider the value of ϵ to be small, that is to say, $\epsilon \ll 1$. From the equations which θ_0 and θ_c satisfy, we obtain the equation for $\eta(= \theta_c - \theta_0)$ as [13]

$$\sin(2\theta_c)(\epsilon - \frac{3}{2}\eta^2) - \eta \cos(2\theta_c)(2\epsilon - \eta^2) = 0. \quad (15)$$

Simple calculations show that η is of the order of $\sqrt{\epsilon}$. Thus the order of magnitude of the second term in Eq. (15) is smaller than that of the first term by $\sqrt{\epsilon}$ and the value of η is determined by the first term which leads to $\eta \simeq \sqrt{2\epsilon/3}$. However, when θ_H is very close to $\pi/2$ or π , $\sin(2\theta_c)$ becomes almost zero, as shown in Fig. 1, and the first term is much smaller than the second term in Eq. (15). Therefore the value of η is obtained from the second term when $\theta_H \simeq \pi/2$ or π , which leads to $\eta \simeq \sqrt{2\epsilon}$ for $\theta_H \simeq \pi/2$ and $\eta \simeq 0$ for $\theta_H \simeq \pi$ from the detailed analysis of the equations which θ_0 and θ_c satisfy. Since the first term in (15) is dominant in the range of value θ_c which satisfies $\tan(2\theta_c) > O(\sqrt{\epsilon})$, $\eta \simeq \sqrt{2\epsilon/3}$ is valid for $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$ by using Eq. (14). As is shown in Fig. 2, this is checked by performing the numerical calculation for the equations which θ_0 and θ_c satisfy. From Eq. (11) we obtain an approximate form of $\bar{E}(\theta, \phi)$ as

$$\begin{aligned} \bar{E}(\delta, \phi) = & \bar{K}_2 \sin^2 \phi \sin^2(\theta_0 + \delta) + \bar{H}_x \sin(\theta_0 + \delta)(1 - \cos \phi) \\ & + \bar{C}[(\nabla \delta)^2 + (\nabla \phi)^2 \sin^2(\theta_0 + \delta)] + \bar{E}_1(\delta), \end{aligned} \quad (16)$$

where $\bar{E}(\theta, \phi)$ is written as $\bar{E}(\delta, \phi)$ by introducing a small variable $\delta = \theta - \theta_0$, and $\bar{E}_1(\delta)$ is a function of only δ given by

$$\bar{E}_1(\delta) = \frac{1}{4} \sin(2\theta_c)(3\delta^2\eta - \delta^3) + \frac{1}{2} \cos(2\theta_c)[\delta^2(\epsilon - \frac{3}{2}\eta^2) + \delta^3\eta - \frac{\delta^4}{4}]. \quad (17)$$

As previously considered for η in Eq. (15), even though the $\cos(2\theta_c)$ -term in Eq. (17) looks smaller by a factor of η which is of the order of $\sqrt{\epsilon}$, the second term can not be neglected

near $\theta_H = \pi/2$ and π because $\sin(2\theta_c)$ is almost zero for these regions of θ_H . In order to evaluate the order of magnitude of each term in the action (5) and simplify the calculations for $\epsilon \ll 1$, it is convenient to use new scaled variables

$$\tilde{\tau} = \epsilon^{\frac{\alpha}{2}} \omega_0 \tau, \quad \tilde{\mathbf{r}} = \epsilon^{\frac{\beta}{2}} \mathbf{r}/r_0, \quad \bar{\delta} = \delta/\sqrt{\epsilon}, \quad r_0 = \sqrt{2\bar{C}}, \quad \omega_0 = 2\gamma K_1/M_0, \quad (18)$$

where it is noted that r_0 is the thickness of the domain wall in a bulk ferromagnet. Then, the Euclidean action (5) becomes

$$\begin{aligned} S_E[\delta(\tilde{\mathbf{r}}, \tilde{\tau}), \phi(\tilde{\mathbf{r}}, \tilde{\tau})] = & \hbar J \epsilon^{-\frac{\alpha+3\beta}{2}} r_0^3 \int d\tilde{\tau} d^3\tilde{\mathbf{r}} \{ i\epsilon^{\frac{\alpha}{2}} [1 - \cos(\theta_0 + \sqrt{\epsilon}\bar{\delta})] \frac{\partial\phi}{\partial\tilde{\tau}} \\ & + \bar{K}_2 \sin^2 \phi \sin^2(\theta_0 + \sqrt{\epsilon}\bar{\delta}) + \bar{H}_x \sin(\theta_0 + \sqrt{\epsilon}\bar{\delta})(1 - \cos \phi) \\ & + \frac{1}{2} \epsilon^{1+\beta} (\tilde{\nabla}\bar{\delta})^2 + \frac{1}{2} \epsilon^\beta \sin^2(\theta_0 + \sqrt{\epsilon}\bar{\delta}) (\tilde{\nabla}\phi)^2 \\ & + \frac{1}{4} \sin(2\theta_c) \epsilon^{\frac{3}{2}} (3\bar{\delta}^2 \frac{\eta}{\sqrt{\epsilon}} - \bar{\delta}^3) + \frac{1}{2} \cos(2\theta_c) \epsilon^2 [\bar{\delta}^2 (1 - \frac{3\eta^2}{2\epsilon}) + \bar{\delta}^3 \frac{\eta}{\sqrt{\epsilon}} - \frac{\bar{\delta}^4}{4}] \}. \end{aligned} \quad (19)$$

where the parameters α and β are fixed by the analysis of the order of magnitude of each term in the action (19) subject to the situation considered.

For $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$, the critical angle θ_c and the angle of metastable state θ_0 are in the range of angle from 0 to $\pi/2$, and $\eta \simeq \sqrt{2\epsilon/3}$. In this situation, the action (19) becomes

$$\begin{aligned} S_E[\delta(\tilde{\mathbf{r}}, \tilde{\tau}), \phi(\tilde{\mathbf{r}}, \tilde{\tau})] = & \hbar J \epsilon^{-\frac{\alpha+3\beta}{2}} r_0^3 \int d\tilde{\tau} d^3\tilde{\mathbf{r}} \{ -i\epsilon^{\frac{\alpha+1}{2}} \sin(\theta_0 + \sqrt{\epsilon}\bar{\delta}) \phi \left(\frac{\partial\bar{\delta}}{\partial\tilde{\tau}} \right) \\ & + \bar{K}_2 \sin^2 \phi \sin^2(\theta_0 + \sqrt{\epsilon}\bar{\delta}) + 2\bar{H}_x \sin(\theta_0 + \sqrt{\epsilon}\bar{\delta}) \sin^2\left(\frac{\phi}{2}\right) \\ & + \frac{1}{2} \epsilon^{1+\beta} (\tilde{\nabla}\bar{\delta})^2 + \frac{1}{2} \epsilon^\beta \sin^2(\theta_0 + \sqrt{\epsilon}\bar{\delta}) (\tilde{\nabla}\phi)^2 \\ & + \frac{1}{4} \sin(2\theta_c) \epsilon^{\frac{3}{2}} (3\bar{\delta}^2 \frac{\eta}{\sqrt{\epsilon}} - \bar{\delta}^3) \}, \end{aligned} \quad (20)$$

where we performed the integration by part for the first term and neglected the total time derivative. Since the last term in the action (20) makes the potential barrier in our problem, the order of magnitude of each term should be the same order as that of the last term. Since the order of magnitude of each term is $O(\epsilon^{(\alpha+1)/2}\phi)$, $O(\sin^2 \phi)$, $O(\sin^2(\phi/2))$, $\epsilon^{1+\beta}$, $O(\epsilon^\beta \phi^2)$

and $\epsilon^{3/2}$, respectively, the small values of ϕ contribute to the path integral for $\epsilon \ll 1$, and its order of magnitude is expected to be $\epsilon^{3/4}$. Choosing $\alpha = \beta = 1/2$, the order of magnitude of the term which includes $(\tilde{\nabla}\phi)^2$ is ϵ^2 while those of the other terms are $\epsilon^{3/2}$. Thus, the fifth term in Eq. (20) is higher and neglected.

Performing the Gaussian integration over ϕ in the transition amplitude (1) and the measure (3), the remaining integral is of the form

$$\int D[\delta(\tilde{\mathbf{r}}, \tilde{\tau})] \exp(-S_E^{eff}/\hbar), \quad (21)$$

where from Eq. (20) the effective action is given by

$$S_E^{eff}[\delta(\tilde{\mathbf{r}}, \tilde{\tau})] = \hbar J r_0^3 \epsilon^{\frac{1}{2}} \int d\tilde{\tau} d^3\tilde{\mathbf{r}} \left[\frac{1}{2} M \left(\frac{\partial \bar{\delta}}{\partial \tilde{\tau}} \right)^2 + \frac{1}{2} (\tilde{\nabla} \bar{\delta})^2 + \frac{1}{4} \sin(2\theta_c) (\sqrt{6} \bar{\delta}^2 - \bar{\delta}^3) \right]. \quad (22)$$

Here the effective mass in the reduced dimension is given by

$$M = \frac{\sin \theta_c}{\bar{H}_x + 2\bar{K}_2 \sin \theta_c}. \quad (23)$$

Introducing the variables $\bar{\tau} = \tilde{\tau} \sqrt{\sin(2\theta_c)/M}$ and $\bar{\mathbf{r}} = \tilde{\mathbf{r}} \sqrt{\sin(2\theta_c)}$, we simplify the effective action (22) to be

$$S_E^{eff}[\delta(\bar{\mathbf{r}}, \bar{\tau})] = \hbar J r_0^3 \epsilon^{\frac{1}{2}} \frac{\sqrt{M}}{\sin(2\theta_c)} \int d\bar{\tau} d^3\bar{\mathbf{r}} \left[\frac{1}{2} \left(\frac{\partial \bar{\delta}}{\partial \bar{\tau}} \right)^2 + \frac{1}{2} (\bar{\nabla} \bar{\delta})^2 + \frac{1}{4} (\sqrt{6} \bar{\delta}^2 - \bar{\delta}^3) \right], \quad (24)$$

In case of a small magnetic particle of volume $V \ll r_0^3$, $\bar{\delta}$ does not depend on the space $\bar{\mathbf{r}}$, which leads to the action

$$S_E^{eff}[\delta(\bar{\tau})] = \hbar J \epsilon^{\frac{5}{4}} V \sqrt{M \sin(2\theta_c)} \int d\bar{\tau} \left[\frac{1}{2} \left(\frac{d\bar{\delta}}{d\bar{\tau}} \right)^2 + \frac{1}{4} (\sqrt{6} \bar{\delta}^2 - \bar{\delta}^3) \right]. \quad (25)$$

The corresponding classical trajectory is given by

$$\bar{\delta}_d(\bar{\tau}) = \frac{\sqrt{6}}{\cosh^2\left(\left(\frac{3}{32}\right)^{1/4} \bar{\tau}\right)}, \quad (26)$$

and its classical action is found to be [13]

$$S_{cl} = (\hbar J) \frac{16 \times 6^{1/4}}{5} \epsilon^{5/4} \frac{|\cot \theta_H|^{1/6}}{\sqrt{1 + \frac{K_2}{K_1} (1 + |\cot \theta_H|^{2/3})}}. \quad (27)$$

The geometry of the problem in this paper is the thin film of thickness z_t less than the size $r_0/\epsilon^{1/4}$ of the critical nucleus and its plane is perpendicular to the initial easy axis. After performing the integration over \bar{z} -variable, the action (24) becomes

$$S_E^{eff}[\delta(\bar{\mathbf{r}}, \bar{\tau})] = \hbar J r_0^2 z_t \epsilon^{\frac{3}{4}} \sqrt{\frac{M}{\sin(2\theta_c)}} \int d\bar{\tau} d^2\bar{\mathbf{r}} \left[\frac{1}{2}(\partial_{\bar{\tau}}\bar{\delta})^2 + \frac{1}{2}(\partial_{\bar{x}}\bar{\delta})^2 + \frac{1}{2}(\partial_{\bar{y}}\bar{\delta})^2 + \frac{1}{4}(\sqrt{6}\bar{\delta}^2 - \bar{\delta}^3) \right], \quad (28)$$

At zero temperature the classical solution of the action (28) has $O(3)$ symmetry in $\bar{\tau}$ and $\bar{\mathbf{r}}$. Therefore, $\bar{\delta}$ is a function of q where $q = (\bar{x}^2 + \bar{y}^2 + \bar{\tau}^2)^{1/2}$, in which the effective action (28) becomes

$$S_E^{eff}[\delta(\bar{\mathbf{r}}, \bar{\tau})] = 4\pi\hbar J r_0^2 z_t \epsilon^{\frac{3}{4}} f(\theta_H) \int_0^\infty dq q^2 \left[\frac{1}{2} \left(\frac{d\bar{\delta}}{dq} \right)^2 + \frac{1}{4} (\sqrt{6}\bar{\delta}^2 - \bar{\delta}^3) \right], \quad (29)$$

with the angular dependence

$$f(\theta_H) = \frac{1 + |\cot \theta_H|^{2/3}}{\sqrt{2} |\cot \theta_H|^{1/6} \sqrt{1 - \epsilon + \frac{K_2}{K_1} (1 + |\cot \theta_H|^{2/3})}}. \quad (30)$$

The corresponding classical equation of motion satisfies

$$\frac{d^2\bar{\delta}}{dq^2} + \frac{2}{q} \frac{d\bar{\delta}}{dq} = \frac{\sqrt{6}}{2} \bar{\delta} - \frac{3}{4} \bar{\delta}^2. \quad (31)$$

This equation can be solved by the numerical approach, whose results are similar to the ones in Ref. [2]. The maximal value of $\bar{\delta}$ is 6.85 at $\bar{\tau} = 0$ and $\bar{r} = 0$. Using the results, we obtain the rate of quantum nucleation given by

$$\Gamma_Q \propto \exp(-S_E/\hbar) = \exp[-105.2 J r_0^2 f(\theta_H) z_t \epsilon^{3/4}]. \quad (32)$$

Since the period τ_I of the instanton becomes zero for the temperature much larger than T_c , the instanton solution becomes independent of $\bar{\tau}$. In this case we have the rate of thermal nucleation

$$\Gamma_T \propto \exp\left(-\frac{U^{eff}}{k_B T}\right) \quad (33)$$

where the effective height of the barrier

$$U^{eff} = 4\pi K_1 r_0^2 z_t \epsilon \int_0^\infty dq q \left[\frac{1}{2} \left(\frac{d\bar{\delta}}{dq} \right)^2 + \frac{\sqrt{6}}{4} \bar{\delta}^2 - \frac{1}{4} \bar{\delta}^3 \right]. \quad (34)$$

From the saddle point of the functional the shape of thermal nucleus is determined by

$$\frac{d^2 \bar{\delta}}{dq^2} + \frac{1}{q} \frac{d\bar{\delta}}{dq} - \frac{\sqrt{6}}{2} \bar{\delta} + \frac{3}{4} \bar{\delta}^2 = 0. \quad (35)$$

The solution is found by the numerical method whose shape is similar to the one in Ref. [2]. The maximal size of the thermal nucleus is 3.9 at $\bar{\mathbf{r}} = 0$. Using this result, the effective height of the barrier becomes

$$U^{eff} = 41.3 K_1 r_0^2 z_t \epsilon. \quad (36)$$

Comparing this with Eq. (32), we obtain the approximate form of the temperature of the crossover from thermal to quantum regime given by

$$T_c \approx 0.22 \frac{K_1 \epsilon^{1/4}}{k_B J f(\theta_H)}. \quad (37)$$

Here we notice that the angular and the field dependence of T_c coincides with the one of Ref. [13] and with that of Ref. [8] in the limit of $\bar{K}_2 = 0$. It is understood from the fact that $T_c \sim \hbar \omega_b / k_B$, where the frequency ω_b of small oscillations around the minimum of the inverted potential is proportional to $\omega_0 \epsilon^{1/4} / f(\theta_H)$ from Eq. (18) and the scaling relation between $\bar{\tau}$ and $\tilde{\tau}$.

Let us now consider the situation with the magnetic field opposite to the initial easy axis. For $\theta_H = \pi$, we obtain $\theta_c = 0$ and $\eta = 0$ by using Eqs. (14) and (15). In this case, noting that $\theta_0 = 0$, $\bar{H}_x = 0$ and $\epsilon \ll 1$, the Euclidean action (19) can be simplified to be

$$\begin{aligned} S_E[\delta(\tilde{\mathbf{r}}, \tilde{\tau}), \phi(\tilde{\mathbf{r}}, \tilde{\tau})] = & \hbar J \epsilon^{-\frac{\alpha+3\beta}{2}} r_0^3 \int d\tilde{\tau} d^3 \tilde{\mathbf{r}} \left[\frac{i}{2} \epsilon^{1+\frac{\alpha}{2}} \bar{\delta}^2 \frac{\partial \phi}{\partial \tilde{\tau}} + \bar{K}_2 \epsilon \bar{\delta}^2 \sin^2 \phi \right. \\ & \left. + \frac{1}{2} \epsilon^{1+\beta} (\tilde{\nabla} \bar{\delta})^2 + \frac{1}{2} \epsilon^{1+\beta} \bar{\delta}^2 (\tilde{\nabla} \phi)^2 + \frac{1}{2} \epsilon^2 \left(\bar{\delta}^2 - \frac{\bar{\delta}^4}{4} \right) \right]. \end{aligned} \quad (38)$$

In order to obtain the quantum tunneling under the barrier, we need the second term which includes the transverse component of the anisotropy energy E_a . Without K_2 in E_a , M_z , as a quantum operator, commute with E_a and conserves. Thus, K_2 is responsible for

quantum transition at $\theta_H = \pi$. Comparing the second term with the last term which makes the potential barrier, only small values of ϕ contribute to the path integral because $\epsilon \ll 1$. Thus we expect the order of magnitude of ϕ to be $\epsilon^{1/2}$. Choosing $\alpha = \beta = 1$, the order of magnitude of each term except the fourth term in Eq. (38) is ϵ^2 while the fourth one is ϵ^3 which is higher and neglected. Following the same procedure as we have done previously, we obtained the WKB exponent $B_Q(T = 0, \theta_H = \pi)$, the effective height of the barrier $U^{eff}(\theta_H = \pi)$ for $T > T_c$ and the approximate form of the crossover temperature T_c , which were presented by Chudnovsky and Gunther [2]. Recently, Ferrera and Chudnovsky discussed the thermally assisted quantum nucleation at $\theta_H = \pi$ and obtained the temperature dependence of the WKB exponent $B(T, \theta_H = \pi)$ in the range of temperature $0 < T < 2T_c$ by numerically solving the partial differential equation with period $\hbar\omega\epsilon^{1/2}/T$ where $\omega = (2\gamma/M_0)\sqrt{K_1K_2}$.

As summarized in Table I, B_Q is proportional to $\epsilon^{3/4}$ for a range of angles $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$, while $B_Q \propto \epsilon^{1/2}$ at $\theta_H = \pi$. By using the interpolation method, we obtain B_Q in the range of angle close to π , and present the results in Fig. 3 for $\epsilon = 0.01$. In case of volume $V \ll r_0^3$ [8,13], B_Q can be minimum at $\theta_H = \pi$ depending on the ratio K_2/K_1 of the anisotropy constants. $B_Q(\theta_H = \pi)$ is minimum for $K_2 = K_1$ while it is maximum in the limit of $K_2 \ll K_1$ which is uniaxial symmetry [8]. In the ferromagnetic film which we have considered with small ϵ , B_Q has a minimum in the range of angle $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$ for both $K_2 = K_1$ and $K_2 \ll K_1$, as is shown in Table I. Also, in the limit of $K_2 \gg K_1$ the dependence of $f(\theta_H)$ on the ratio K_2/K_1 is the same as that of B_Q at $\theta_H = \pi$. Thus, in the small ϵ limit, the ratio does not make an effect on the shape of B_Q , which is different from the situation for $V \ll r_0^3$. [13] In other words, since B_Q has a shape similar to Fig. 3, it always has a minimum in the range of angle $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$. The minimal value of B_Q or equivalently the maximal value of Γ_Q at $T = 0$ can be obtained by the analysis of $f(\theta_H)$ and its θ_H is given by

$$\theta_H = \pi - \arctan\left[\frac{2\bar{K}_2}{\sqrt{(2\bar{K}_2 + \frac{1}{2})^2 + 2 - \frac{3}{2}}}\right]^{3/2}, \quad (39)$$

where we notice that $\theta_H \approx 1.97 (\simeq 112.9^\circ)$ for, e.g., $K_1 = K_2$, as is shown in Fig. 3. Simple analysis shows that the angle which gives the minimal value of B_Q , Eq. (39), decreases as \bar{K}_2 decreases, and converges to $\pi - \arctan(3\sqrt{3}) (\approx 1.76 \simeq 100.9^\circ)$ in the limit of $\bar{K}_2 = 0$. [8] Since the effective height of barrier U^{eff} is proportional to ϵ for both $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$ and $\theta_H = \pi$, we obtain $T_c \propto \epsilon^{1/4}$ for $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$ and $T_c \propto \epsilon^{1/2}$ for $\theta_H = \pi$. Since T_c is inversely proportional to $f(\theta_H)$ for a wide range of angles, T_c becomes maximum at the angle (39). For $K_1 \simeq K_2 \simeq 10^7 \text{ erg/cm}^3$, $M_0 \simeq 500 \text{ emu/cm}^3$ and $\epsilon \sim 10^{-2} - 10^{-3}$, the maximal value of T_c is approximately $235 - 132 \text{ mK}$ while $T_c \approx 167 - 53 \text{ mK}$ for $\theta_H = \pi$.

In conclusion, we have studied the quantum nucleation of a thin ferromagnetic film placed at some angle with the magnetic field and presented the quantum nucleation exponent and the crossover temperature based on the spin coherent path integral formalism. If the experiment is to be performed, there are three control parameters for comparison with theory: the angle of the magnetic field θ_H , the magnitude of the field \mathbf{H} in terms of ϵ , and the temperature T . The dependence of B_Q and T_c on the power of ϵ seems to be observed by changing θ_H . Also, the angle (39) which gives the maximal value of T_c as well as Γ_Q is expected to be observed in experiment.

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TABLES

TABLE I. Summary of the results for the quantum nucleation in the range of angle $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$ and $\theta_H = \pi$. Here B_Q is the WKB exponent at $T = 0$ and $f(\theta_H)$ given by Eq. (30).

Field angle	$\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$	$\theta_H = \pi$ [2]
$\alpha = \beta$	$1/2$	1
M	$(2\bar{K}_2 + \bar{H}_x / \sin \theta_c)^{-1}$	$(4\bar{K}_2)^{-1}$
$B_Q / Jr_0^2 z_t$	$105.2 f(\theta_H) \epsilon^{3/4}$	$37.9 \sqrt{K_1/K_2} \epsilon^{1/2}$
$U^{eff} / K_1 r_0^2 z_t$	41.3ϵ	23.4ϵ
$(k_B T_c) J / K_1$	$0.22 \epsilon^{1/4} / f(\theta_H)$	$0.62 \sqrt{K_2/K_1} \epsilon^{1/2}$

FIGURES

FIG. 1. Comparison of (a) $\sin(2\theta_c)$ with (b) $\cos(2\theta_c)$ in Eqs. (15) and (17). Note that $\sin(2\theta_c) = 0$, $|\cos(2\theta_c)| = 1$ for both $\theta_H = \pi/2$ and π .

FIG. 2. The shape of $\eta(= \theta_c - \theta_0)$ as a function of θ_H for (a) $\epsilon = 0.01$ and (b) $\epsilon = 0.001$. Note that $\eta \approx \sqrt{2\epsilon/3}$ is valid in a wide range of angles, i.e., $\pi/2 + O(\sqrt{\epsilon}) < \theta_H < \pi - O(\sqrt{\epsilon})$.

FIG. 3. The θ_H dependence of the quantum nucleation exponent \bar{B}_Q for $K_1 = K_2$ and $\epsilon = 0.01$, where $\bar{B}_Q = B_Q/Jr_0^2 z_t$. The minimal value of \bar{B}_Q is approximately 2.65 at $\theta_H \approx 1.97$ while $\bar{B}_Q \approx 3.79$ at $\theta_H = \pi$.





